Math identities and other results

Gaussian integrals:

$$I^{(0)}(a,b) \equiv \int_{-\infty}^{+\infty} e^{-ax^2-bx}\,dx = \sqrt{\pi/a}\exp(b^2/4a)$$
$$I^{(1)}(a,b) \equiv \int_{-\infty}^{+\infty} xe^{-ax^2-bx}\,dx = (-2b/a)\sqrt{\pi/a}\exp(b^2/4a)$$
$$I^{(2)}(a,b) \equiv \int_{-\infty}^{+\infty} x^2e^{-ax^2-bx}\,dx = [(b^2+2a)/4a^2]\sqrt{\pi/a}\exp(b^2/4a)$$

Other integrals:

$$\int \sin^2(ax)\,dx = x/2 - \sin(2ax)/4a$$
$$\int \cos^2(ax)\,dx = x/2 + \sin(2ax)/4a$$
$$\int \sin(mx)\sin(nx)\,dx = +\sin[(m-n)x]/2(m-n) - \sin[(m+n)x]/2(m+n)$$
$$\int \cos(mx)\cos(nx)\,dx = +\sin[(m-n)x]/2(m-n) + \sin[(m+n)x]/2(m+n)$$
$$\int \sin(mx)\cos(nx)\,dx = -\cos[(m-n)x]/2(m-n) - \cos[(m+n)x]/2(m+n)$$

Trig identities:

$$\begin{aligned}
\sin(a\pm b) &= \sin(a)\cos(b) \pm \cos(a)\sin(b)\\
\cos(a\pm b) &= \cos(a)\cos(b) \mp \sin(a)\sin(b)\\
\sin(a)\sin(b) &= \cos(a-b)/2 - \cos(a+b)/2\\
\cos(a)\cos(b) &= \cos(a-b)/2 + \cos(a+b)/2\\
\sin(a)\cos(b) &= \sin(a-b)/2 + \sin(a+b)/2\\
\cos(a)\sin(b) &= \sin(a+b)/2 - \sin(a-b)/2\\
\exp(\pm iz) &= \cos(z) \pm i\sin(z)
\end{aligned}$$

Free-particle Gaussian wave packets (Chapters 3 and 4):

$$\phi_{(G)}(p,t) = \phi_0(p)e^{-ip^2t/2m\hbar} = \left[\sqrt{\frac{\alpha}{\sqrt{\pi}}}\,e^{-\alpha^2(p-p_0)^2/2}e^{-ipx_0/\hbar}\right]e^{-ip^2t/2m\hbar}$$
$$\psi_{(G)}(x,t) = \frac{1}{\sqrt{\sqrt{\pi}\alpha\hbar(1+it/t_0)}}\,e^{ip_0(x-x_0)/\hbar}e^{-ip_0^2t/2m\hbar}\,e^{-(x-x_0-p_0t/m)^2/2(\alpha\hbar)^2(1+it/t_0)}$$
$$|\psi_{(G)}(x,t)|^2 = \frac{1}{\sqrt{\pi}\beta_t}e^{-(x-x_0-p_0t/m)^2/\beta_t^2}$$
$$\langle p\rangle_t = p_0, \quad \langle p^2\rangle_t = p_0^2 + \frac{1}{2\alpha^2}, \quad \text{and} \quad \Delta p_t = \Delta p_0 = \frac{1}{\alpha\sqrt{2}}$$
$$\langle x\rangle_t = x(t) \equiv x_0 + p_0t/m, \quad \langle x^2\rangle_t = [x(t)]^2 + \frac{\beta_t^2}{2}, \quad \text{and} \quad \Delta x_t = \frac{\beta_t}{\sqrt{2}}$$

where

$$\beta_t \equiv \alpha\hbar\sqrt{1+t^2/t_0^2} \quad \text{and} \quad t_0 \equiv m\hbar\alpha^2 = \frac{m\hbar}{2(\Delta p_0)^2} = \frac{2m(\Delta x_0)^2}{\hbar}$$